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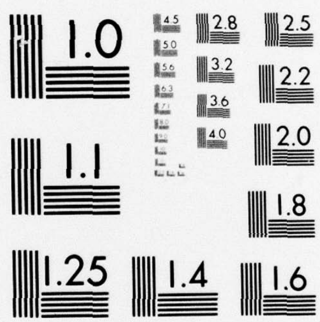

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The possibility of aligning the dual goals of an optimal stochastic controller is discussed. It is suggested that when the measurement function is chosen so that these two dual goals are aligned an artificial separation will occur. This will occur since the action taken to follow a trajectory will also lead to the best possible use of the control for estimation purposes. A simple set of examples describing the nature of such alignment and cases of nonalignment is given.

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NONLINEAR FILTERS IN FEEDBACK CONTROL*

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Abstract

The possibility of aligning the dual goals of an optimal stochastic controller is discussed. It is suggested that when the measurement function is chosen so that these two dual goals are aligned an artificial separation will occur. This will occur since the action taken to follow a trajectory will also lead to the best possible use of the control for estimation purposes. A simple set of examples describing the nature of such alignment and cases of nonalignment is given.

1. INTRODUCTION

Inherent in the problem of stabilization and control of most dynamic systems, is the problem of processing noise contaminated measurement data to obtain accurate information about the state of the generally nonlinear stochastic system. If the state can be accurately estimated, then classical deterministic control techniques, or an approximate linearized quadratic gaussian approach, can often be used to give adequate system performance. The classical deterministic controller is often of the form of a feedback control scheme.[1,2] As pointed out by Feldbaum,[3] the optimal stochastic controller for nonlinear stochastic systems can often be thought of as having two (possibly conflicting) goals. The first goal is to drive the "true" state of the system over or near a desired trajectory in state space. The second goal is to obtain the most accurate information about the value of the "true" state.

The desired trajectory is usually specified in terms of a cost functional. One measure of the cost of a trajectory is its deviation from the desired trajectory. This cost is added to a cost for the control action required to traverse the trajectory. The "optimal" trajectory is the one which minimizes the combined cost found by adding these two.

2. TWO GOALS

If there are no uncertain system parameters, and no noise driving the dynamic system or corrupting the measurements, then, the optimal feedback control at time t_k is a function of the state at time t_k.

$$\underline{U}(t_k) = \varphi(\underline{x}_k, t_k) \qquad 0 \le t \le T \qquad (1)$$

or

$$\underline{U}_k = \varphi_k(x_k) \qquad k = 1, 2, \ldots N \qquad (2)$$

The cost is a functional of the control policy $U_1, U_2, \ldots U_N$.

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$$C = g[x_1, x_2, \ldots x_N,$$
$$U_1(x_1), U_2(x_2), \ldots U_N(x_N)] \quad (3)$$

and the optimal control policy is the set of functions $U_1(\ )$, $U_2(\ )$, ... $U_N(\ )$ which minimize the cost.

However if there are uncertain system parameters, noise driving the dynamic system and/or noise contaminating the measurements, the "true" system state is a random variable as is the cost. It is generally impossible to know its present value or to predict its future value with certainty. Our knowledge of the state of the system at the present time is obtained by filtering the measurement data in order to obtain the best possible estimate for state at time k based on all data up to time t_k, $(\hat{x}_{k/k})$.

It should be clear that more accurate estimates of the present state, and prediction of the future state of the system, will allow better control of that state. In general the control policy can affect the accuracy of the estimates and predictions. This use of the control policy to improve state estimation and prediction is the second "dual" goal of the control policy.

A control policy that has seen much use is simply the use of the best estimate for the system state in place of the "true" but unavailable state in a deterministic control policy. This policy is called the certainty equivalence policy. In terms of the policy shown in Eq. (2), the certainty equivalence policy would be written

$$U_{CE}(k) = \varphi_k(\hat{x}_{k/k}) .$$

However, since, as pointed out above, the control policy in general affects the quality of the estimate for the system state, it is possible that in order to observe the system better, the control should be driven in a direction different from the one which would be optimal if there were no noise contaminating the measurements or driving the system.

Note that a certainty equivalence control policy, by its very nature, neglects this second "dual" goal of the optimal stochastic control policy. By definition the control function used in the certainty equivalence policy, $\varphi_k(\ )$, is the optimal policy for the deterministic control problem obtained by replacing all of the noise terms and unknown parameters by the mean values. The certainty equivalence control policy $[U_{CE}(k), k = 1, N]$ thus spends all of its energy in trying to satisfy the first goal, keeping the state on the desired trajectory with a minimum of control energy. This is done ignoring the uncertainty in the knowledge of the true value of the system state and at the expense of the second goal of learning more about the true value of the state. Since the knowledge gained by "probing" the system could enhance the accuracy of the state estimate and thus allow more accurate control, it could greatly aid in minimizing the overall control cost. This discussion should point out why the certainty equivalence policy is generally suboptimal.

3. CAUTION AND PROBING

In an interesting series of papers by Bar-Shalom and Tse[4, 5, 6, 7] there are discussions of the nature of optimal stochastic control policies and certain approximations to such policies. They note (similar to Feldbaum) that in stochastic control systems the optimal stochastic control can have two effects. First it can "probe" the system in order to "look into the future" and make the most use of present information about what might be learned from later measurements. To calculate such a policy in a feedback sense will require considerable mathematical analysis. The second effect they point out is the need for "caution". The "caution" term enters by an apparent increase in the cost of control thereby reducing the amount of control effort that can be used in the optimal system. Both of these effects arise from a detailed study of the structure of the dynamic programing approach (the Principle of Optimality) and both effects are due to the second "dual" effect.

4. TWO GOALS, TWO CONTROL VECTORS

Here a much simpler look at the problem is discussed in a simple tutorial manner. It is hoped that this discussion will lead to additional insight on the part of the reader.

In any given realization, and at any specified time step k, the control required to satisfy either of the two "dual" goals in an optimal manner could be plotted in appropriate control space. The control needed for the first goal (follow the desired trajectory ignoring probabilistic considerations) and the second (obtain the most accurate possible estimate of the system state) need not be conflicting. On the other hand, they could very well be conflicting. In a dynamical system these two control effects could well be in agreement at one stage and in opposition at the next.

The possibilities are indicated in Figure 1 for a single time k. These figures show the possible relationship between the control which would be used, ignoring the ability of the control to affect the estimation accuracy ($\underline{U}_{CE}$), and the control chosen to give the best, minimum variance, estimate without regard to the desired trajectory ($\underline{U}_{EST}$) at time k.

In addition to $\underline{U}_{CE}$ and $\underline{U}_{EST}$ there could be a zero vector on each figure which would represent the correct control vector to use to get minimum control effort. The cost of control represented by this last zero vector is already included in the calculation of the deterministic certainty equivalence control ($\underline{U}_{CE}$). The "estimation optimal" control $\underline{U}_{EST}$ should, perhaps, also be reduced to account for a finite cost for control.

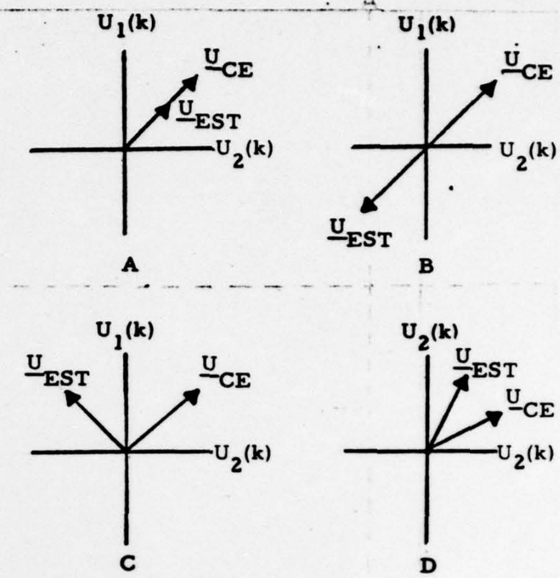

Figure 1

It is enlightening, in certain cases, to think of the optimal stochastic control at stage k in terms of these two control vectors. For example, if the control is incapable of affecting the estimation accuracy, there is no "estimation optimal" control and $\underline{U}_{EST}$ should not appear on the diagram or should be given zero weight. It is just in this case that the "separation theorem" can be derived and the certainty equivalence control is found to be the optimal stochastic control.[8, 9, 10]

At another extreme, one might envision the case where the control could affect the estimation accuracy, but could not affect the basic deterministic cost function itself. In this case, there would be no reason to improve the estimation accuracy and the optimal control policy would be to use no control effort. This latter case shows that the second "dual effect" is secondary to the first.

It should be noted from Figure 1A that the controls required for each of the dual effects do not necessarily have to be different. Thus, in Figure 1A one would use control vectors pointing in the same direction for both effects. If the length of the two vectors is the same at each stage, the control policy giving the "best estimation" (U_{EST}) would be the same as the one ignoring probablistic effects (U_{CE}). If U_{EST} is considerably shorter than U_{CE}, the optimal control could be expected to exhibit some "caution".[5] On the other hand, if the two control vectors were parallel as in Figure 1C but U_{CE} was shorter than U_{EST}, the optimal control would be expected to be stronger than that demanded by U_{CE}. This could be thought of as additional "probing".[5] In an extreme case of "caution" U_{CE} and U_{EST} are actually in opposite directions (Figure 1B). In such a case, the control used for one goal might be the worst thing you could do for the other. A detailed study of the dynamics and stagewise progression of the system would be required, to obtain even an approximation of the true "optimal stochastic control", rather than considerations of the tradeoff of these two controls at each stage. This is because the conflict might appear to demand no action at any given stage but considerations of the effect of the total control policy might indicate that effort should be expended immediately to observe the system so that it could be controlled more accurately at later stages.

The possibility of U_{EST} being orthogonal to U_{CE} is indicated in Figure 1C. While at a single stage, a control which was the vector sum of the two might make sense, the continued use of such a combination (if the orthogonality persisted) could drive the system far from the desired trajectory. Thus, while the orthogonal "probing" required by U_{EST} would affect the optimal control policy, any "probing" actually used would have to be considered later in the trajectory in order to keep the system on the correct path.

From the above discussion it can be seen that these are only two cases where the "optimal stochastic control law" can easily be obtained. The first is when the control cannot affect the estimation accuracy and the generally used certainty equivalence control law is optimal. The second case would be one where U_{EST} is coincident with U_{CE} at each stage. In this case, U_{CE} would again be the optimal control policy. Even if the two controls are only colinear and approximately the same length, we would expect U_{CE} to drive the state vector in such a way that the measurements could be used to adequately estimate the state. Such estimates might involve the use of fairly complex nonlinear filters but there would be no need to consider the dual effect explicitly.

Here we suggest that the choice of the measurement device or measurement function can lead to the aligning of these two vectors in Figure 1. This aligning of the two goals will lead to what we define as a "natural probing" of the system. The designer should choose his measurement structure so that it will estimate the state in the best manner possible when the control system is driving the system to desired trajectory. Then the dual goals will be naturally satisfied without explicit consideration of the effect of the control on the estimation accuracy. This effect is discussed in terms of a simple set of examples below.

5. SIMPLE EXAMPLES

Consider the simple scalar stochastic dynamic control system

$$x_{k+1} = \varphi x_k + U_k + w_k \qquad k, 0, 1, \ldots N \qquad (5)$$

where the scalar control is to be chosen in order to minimize the expected value of the random cost functional C.

$$J = E\left\{C[U_0, U_1(\ ), \ldots U_i(\ )]\right\} \tag{6}$$

$$C = \sum_{j=1}^{N+1} (a_j x_j^2 + b_{j-1} U_{j-1}^2) \tag{7}$$

The noise driving the dynamics is taken to be a white, zero-mean stochastic process with covariance matrix Q_k. The stochastic process w_k is independent of the a priori state x_0 which is a random variable with mean $\hat{x}_0$ and covariance P_0.

The relationship between the controls required to satisfy the dual goals at each stage changes with the choice of measurement function. Consider the following cases.

Case 1

No measurement information.

Case 2

$$z_k = H_k x_k + v_k$$

Case 3

$$z_k = H_k x_k^2 + v_k$$

Case 4

$$z_k = H_k x_k + v_k U_{k-1}^2$$

Case 5

$$z_k = \mathrm{SIGN}(H_k x_k + v_k)$$

In each case v_k is taken to be a zero mean white stochastic process with covariance R_k v_k is also independent of both w_k and x_0.

First remember that if Q_k and P_0 are zero we reduce to the deterministic case and x_k is explicitly available. In this case the optimal deterministic control policy is

$$U_k^D = -\Lambda_k x_k$$

when written as a feedback control policy or

$$U_k^D = -\Lambda_k(\varphi^k x_0 + \varphi^{k-1} U_0 + \varphi^{k-2} U_1 \ldots + U_{k-1})$$

when written as an equivalent open loop policy. The calculation of the Λ_k is well documented in the literature.(8)

In Case 1 and Case 2 the optimal stochastic control policy can be explicitly calculated. In Case 1 it is given by

$$U_k^1 = -\Lambda_k(\varphi^k \hat{x}_0 + \varphi^{k-1} U_0 + \ldots U_{k-1})$$

and in Case 2 the optimal stochastic control policy is given as

$$U_k^2 = -\Lambda_k \hat{x}_{k/k}$$

Here $\hat{x}_{k/k}$ is the best linear estimate of the state x_k conditioned on all the measurement data z_1, z_2, $\ldots z_k$ and is given by the Kalman filter. In both cases the control policy is the same as the optimal deterministic control policy with the state replaced by the best available estimate for state at stage k. These optimal stochastic control policies can be obtained by solving two separate problems, the deterministic control problem and the state estimation problem. This fact is called the Separation Principle. The principle results from the fact that the control policy has no effect on the estimation accuracy. In this case the second control U_{EST} in Figure 1 is indeterminant and should be given no weight.

Case 3 is discussed in references (11) and (12). Due to the nature of the cost function U_{CE} will always try to drive the state estimate to the origin. However, due to the nature of the measurement the signal to noise ratio will be worst at the origin and will get better as the state moves from the origin. Thus U_{CE} will point in the opposite direction from U_{EST}. As shown in reference (12) using U_{CE} at the first stage can be the worst possible control to use. This is the situation indicated in Figure 1B.

In Case 4 (discussed in more generality in refer-

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ence 13), the use of the control at stage k-1 will reduce the accuracy of the measurements at stage k. Since the best measurements are obtained when no control is used, U_{EST} is identically zero. The optimal control is thus reduced by some amount from U_{CE}. As can be seen from this and as shown in reference (13), the optimal stochastic control is again of the form

$$U_k^4 = -\Lambda_k^* \hat{x}_{k/k}$$

but the weighting matrices (Λ_k^*) are not the same as in the deterministic case or in Cases 1 and 2. The control and estimation problem are again separated but the control is not the certainty equivalence control. The use of U_{CE} will increase the noise in the measurements and generally effect the accuracy of the state estimates and thereby greatly degrade the control performance. Here the optimal stochastic control exhibits "caution". The optimal stochastic control is aligned with U_{CE} but reduced in length as it would be in Figure 1A.

In Case 5, discussed at some length in reference (14), the two control goals are aligned. The measurement function adds maximum information about the state when the sign changes unexpectedly or when the "true state" is near zero. The desired trajectory in this problem will also require that the control drive the state to zero (regulator problem). Thus as shown in reference (14), the performance obtained from the use of U_{CE} is very close to a known lower bound to the performance of the optimal stochastic control. Thus in aligning the two control objectives we can approach the true optimal stochastic control ("dual performance") with much less computation than required to calculate the true "dual control" law.

6. CONCLUSIONS

Here it has been suggested that, when dealing with dynamic stochastic systems observed by noisy measurements, when possible, measurement transducers be selected with an eye to aligning the two dual control goals. This could result in improved control system performance without requiring increasingly complicated control laws. The aligning of these two goals will lead to an approximate type of separation principle in that the control chosen for the primary goal will tend to automatically drive the system to improve the accuracy of state estimates.

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